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1990 J. Phys. A: Math. Gen. 23 5089

(http://iopscience.iop.org/0305-4470/23/21/042)

J. Phys. A: Math. Gen. 23 (1990) 5089-5094. Printed in the UK

COMMENT

Numerical simulations of radial displacement of a wetting fluid by a non-wetting fluid in a porous medium

D G Kiriakidis, G H Neale and E Mitsoulis

Department of Chemical Engineering, University of Ottawa, Ottawa, Ontario, K1N 6N5, Canada

Received 26 June 1990, in final form 14 August 1990

Abstract. An approach to simulate the radial displacement of a wetting fluid by a non-wetting fluid in a porous medium is described. The computer algorithm is based on the DLA, anti-DLA and invasion percolation models as well as on the notion of the phase diagram. The transition from DLA and anti-DLA to invasion percolation is made according to a transition probability. The numerical results are in very good qualitative agreement with numerical and experimental results available in the literature.

The three types of immiscible displacement of a wetting fluid by a non-wetting fluid in a porous medium, namely, viscous fingering, capillary fingering and stable displacement are described by the phase diagram [1, 2]. Each type corresponds to a region within the phase diagram having as its axes the viscosity ratio, $M(=\mu_{nw}/\mu_w)$, and the capillary number, $Ca(=V\mu_{nw}/\gamma)$. The boundaries of each region are calculated in terms of the viscosity ratio, the capillary number and the geometrical properties of the porous medium.

Three distinct statistical models have been developed in order to describe the above regions: (*a*) the DLA model (diffusion-limited aggregation) [3, 4] for viscous fingering at low viscosity ratios, (*b*) the anti-DLA model [4] for stable displacement, and (*c*) the invasion percolation model [5] at very low capillary numbers.

Both the DLA and anti-DLA models solve the Laplace equation by letting random walkers wander in the displaced and displacing phases, respectively, and stick upon contact with the interface. The absence of walkers from one phase implies negligible pressure gradients in that phase. According to the invasion percolation model, in the case of drainage the interface moves along the paths of least resistance which are present in the largest channels, since they provide the lowest capillary pressure.

An algorithm has been developed by Leclerc and Neale [6] in order to describe radial, immiscible displacement of a wetting fluid in a porous medium represented by a network of interconnected capillaries. By using the DLA and the anti-DLA models, as well as the notion of open bonds for percolation, they described the transition from viscous fingering to capillary fingering at low viscosity ratios and the transition from stable displacement to capillary fingering at high viscosity ratios. A similar method has been employed by Kiriakidis *et al* [7] to simulate linear displacement of a wetting fluid by a non-wetting one.

Although Leclerc and Neale's algorithm describes successfully the intermediate regions it fails to describe the capillary fingering region at very low capillary numbers. According to their approach one expects an almost complete recovery of the wetting

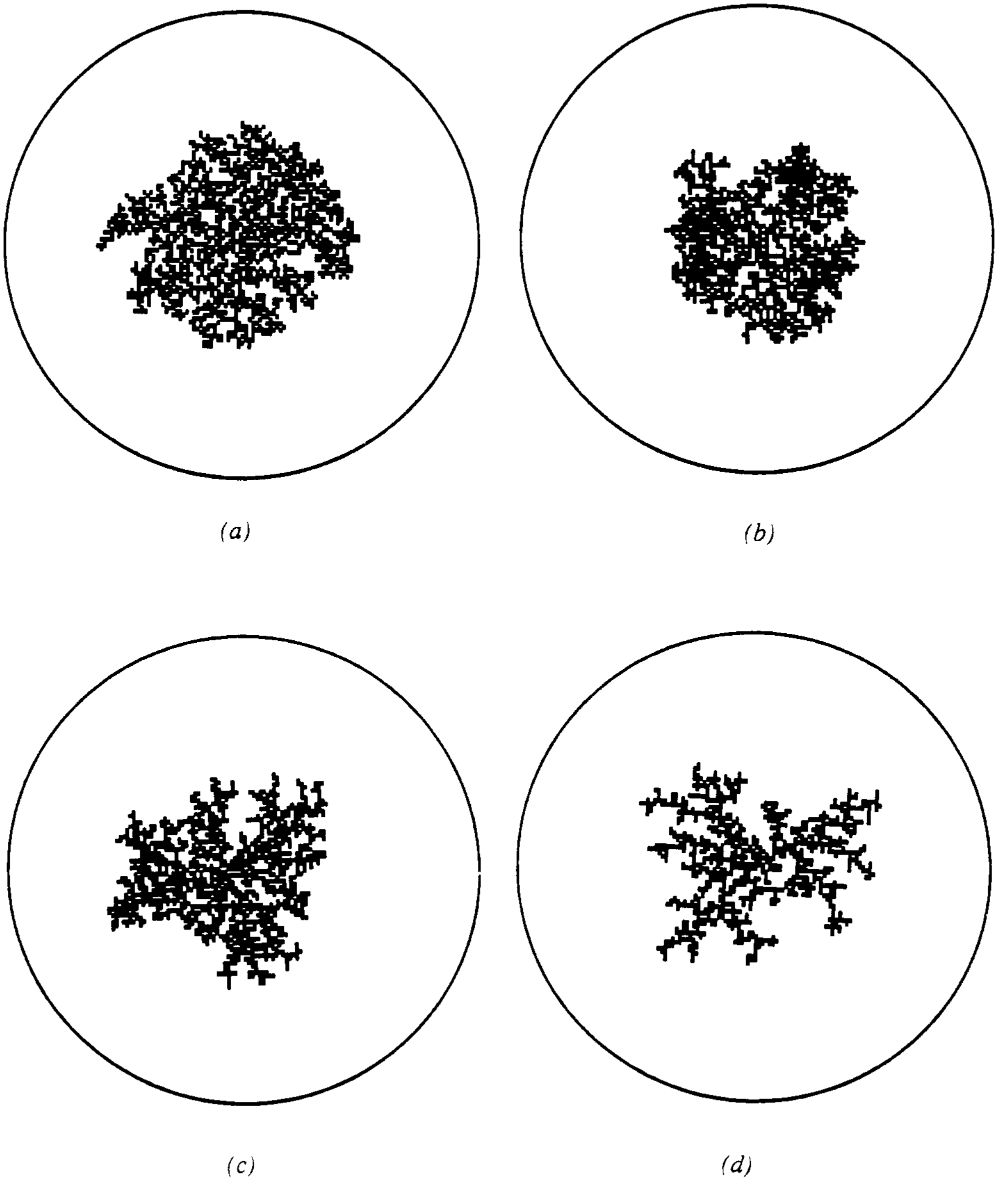
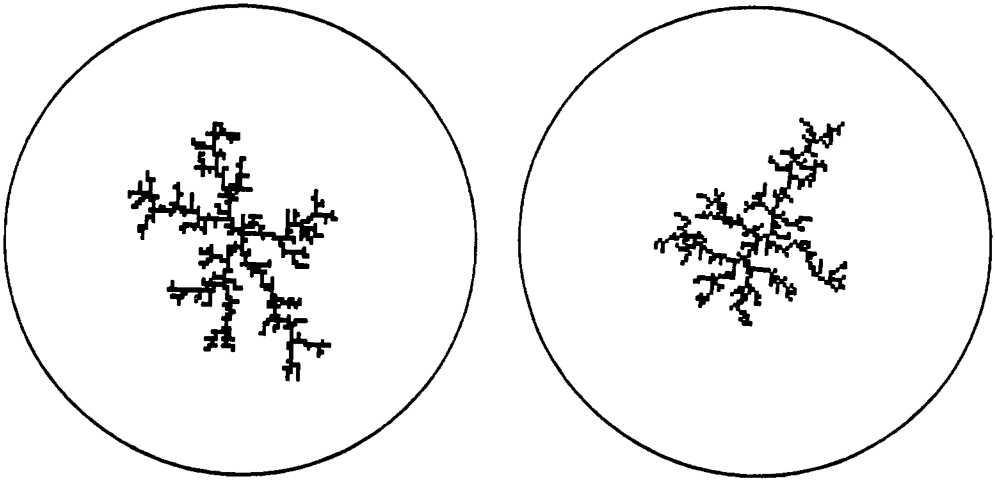


Figure 1. Numerical experiments for $M = 7.6 \times 10^{-5}$ and different capillary numbers: (a) $Ca = 2.30 \times 10^{-11}$, (b) $Ca = 2.30 \times 10^{-10}$, (c) $Ca = 1.15 \times 10^{-9}$, (d) $Ca = 1.15 \times 10^{-8}$, (e) $Ca = 1.15 \times 10^{-7}$, (f) $Ca = 1.15 \times 10^{-6}$.

fluid at very low capillary numbers, since random walkers approach the interface with high sticking probabilities and high numbers of open bonds for percolation. Both experimental and numerical studies [8, 9] confirm the existence of a plateau in the capillary region when the recovery is plotted against the capillary number. The use of non-wetting walkers in the capillary fingering region and at low viscosity ratios is not justified since viscous forces in the displacing non-wetting fluid are negligible compared to capillary forces and to viscous forces in the wetting fluid.



(*e*) (*f*)

Figure 1. (continued)

To resolve the above problems the present authors made use of a different approach. In this approach use is made not only of the DLA and anti-DLA models but also of the invasion percolation model in order to describe displacements in the capillary fingering region. In this region pressure gradients in both phases are negligible compared to the capillary pressure and the interface advances according to the invasion percolation mechanism. The absence of random walkers accounts for negligible pressure gradients in both phases. The notion of the sticking probability is used exactly as in [6] and the boundaries of the phase diagram are calculated from the same equations, i.e. equations (9), (10) and (11) of [6]. No use is made of the notion of open bonds for percolation. According to the present approach, displacements at low viscostiy ratios and for $Ca > Ca_{\mathrm{DLA}}$ are described by the DLA model. In this region only viscous forces in the displaced wetting phase are significant. Within the intermediate region ($Ca_{\mathrm{DLA}} > Ca > Ca_{\mathrm{INV}}$) viscous forces in the displaced phase are comparable to the capillary forces. The interface moves according to both the DLA and the invasion percolation mechanisms, with a 'transition probability' given by

$$\eta = \frac{\mathrm{O}(Ca_{\mathrm{DLA}}) - \mathrm{O}(Ca)}{\mathrm{O}(Ca_{\mathrm{DLA}}) - \mathrm{O}(Ca_{\mathrm{INV}})} \tag{1}$$

where Ca_{DLA}, Ca_{INV} are the capillary numbers at the DLA and invasion percolation limits and 'O' denotes the order of these quantities as approximated by the logarithm. The transition probability is different from the normalized pressure drop used in [6]. It is expressed in terms of the capillary number of the displacement and the limits of the phase diagram without including the viscosity ratio as in [6].

Therefore, in the intermediate region where Ca is close to Ca_{DLA} the interface advances predominantly according to the DLA mechanism. As Ca approaches Ca_{INV} the interface advances according to the invasion percolation mechanism. Finally, for $Ca < Ca_{\mathrm{INV}}$ the interface moves solely according to the invasion percolation mechanism. Within this latter region viscous forces are negligible compared to capillary forces

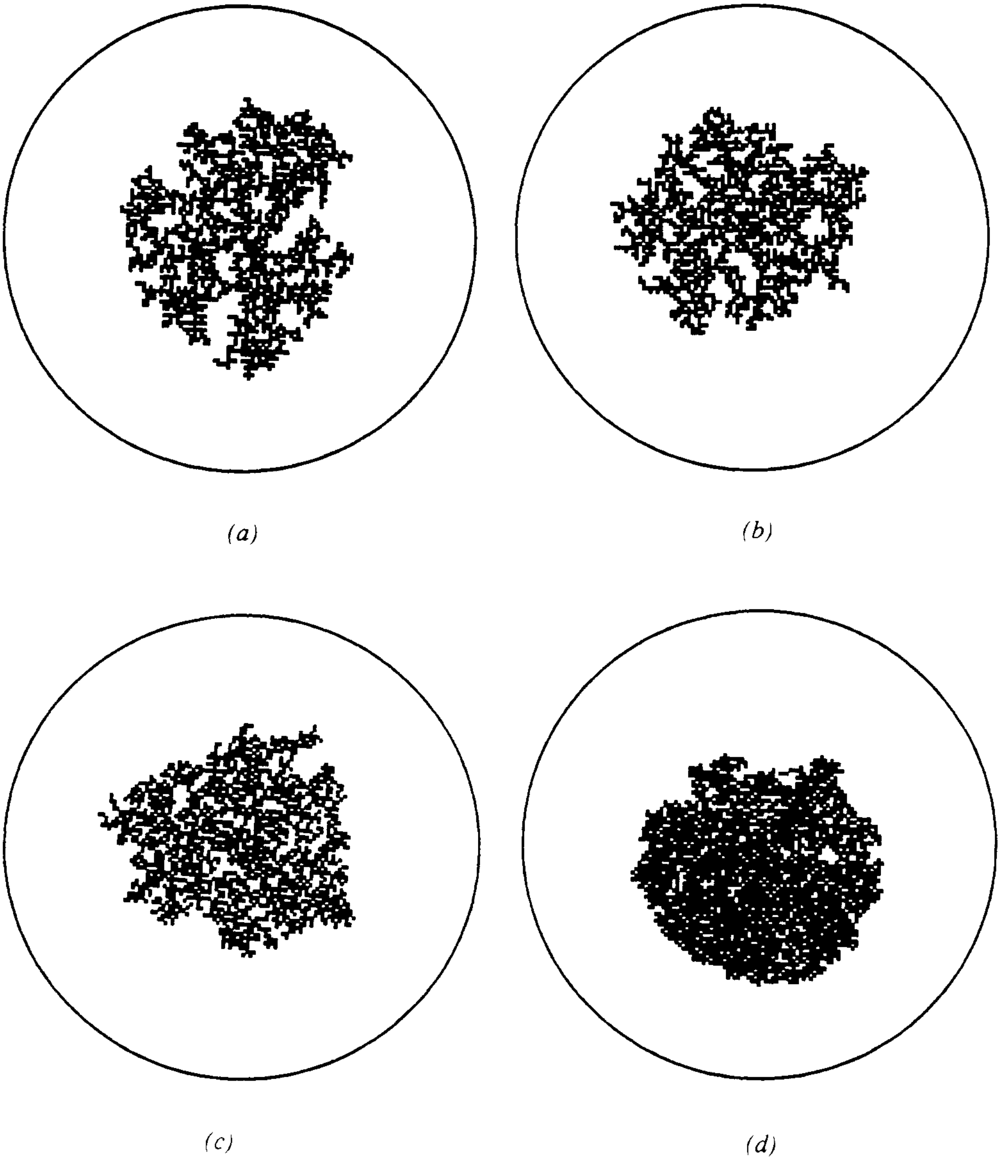


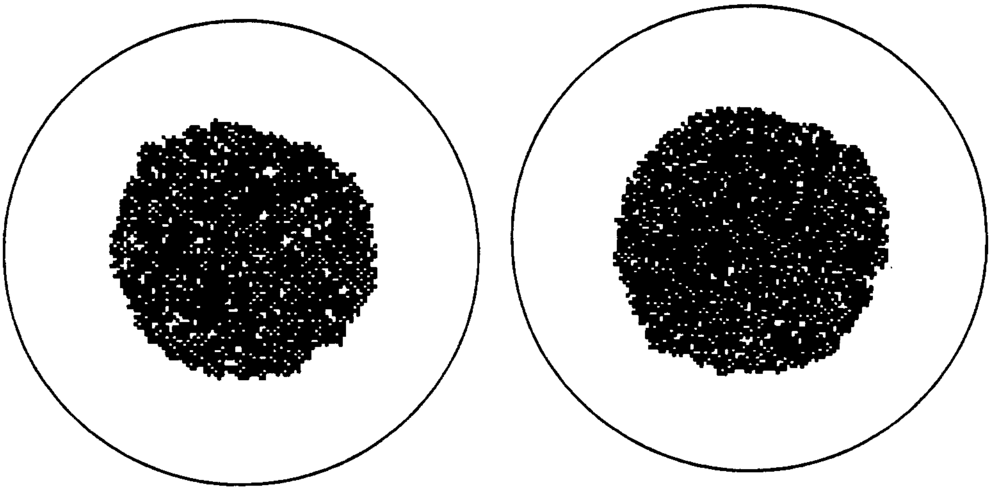
Figure 2. Numerical experiments for $M=13$ and different capillary numbers: (a) $Ca=3.0\times10^{-7}$, (b) $Ca=3.0\times10^{-6}$, (c) $Ca=6.0\times10^{-6}$, (d) $Ca=1.5\times10^{-5}$, (e) $Ca=1.5\times10^{-4}$, (f) $Ca=1.5\times10^{-2}$.

and the interface moves through the largest channels since these channels provide the lowest capillary pressure, P_c, expressed by

$$P_c=\frac{4\gamma\cos\theta}{d} \tag{2}$$

where γ is the interfacial tension, θ the contact angle and d the channel diameter.

The transition from the stable displacement region to the capillary fingering region is described in a similar way. Thus, instead of the DLA model, the anti-DLA model is



(e) (f)

Figure 2. (continued)

used and the frequency ratio of having the anti-DLA mechanism or the invasion percolation mechanism depends on a probability given by an equation similar to (1).

The results of the different transitions are presented in figures 1 and 2. These simulations were performed for the same conditions as those employed by Leclerc and Neale [6] and for the same sets of experimental data as those presented by Lenormand and Zarcone [10]. The fractal dimensions were calculated by using a box-counting algorithm, and are presented in table 1 for reference.

To summarize, an approach has been developed to describe the transition from viscous fingering to capillary fingering at low viscosity ratios, and the transition from stable displacement to capillary fingering at high viscostity ratios. The results of the simulations are in qualitative agreement with both experimental and numerical results available in the literature.

Table 1. Results of the simulations.

Figure	M	Ca	D	% recovery
1*a*	7.6×10^{-5}	2.30×10^{-11}	1.80–1.84	50–55
1*b*	7.6×10^{-5}	2.30×10^{-10}	1.80–1.84	50–55
1*c*	7.6×10^{-5}	1.15×10^{-9}	1.77–1.81	43–47
1*d*	7.6×10^{-5}	1.15×10^{-8}	1.75–1.78	33–39
1*e*	7.6×10^{-5}	1.15×10^{-7}	1.64–1.68	15–18
1*f*	7.6×10^{-5}	1.15×10^{-6}	1.53–1.56	12–14
2*a*	13	3.0×10^{-7}	1.80–1.84	50–55
2*b*	13	3.0×10^{-6}	1.80–1.84	50–55
2*c*	13	6.0×10^{-6}	1.80–1.84	50–55
2*d*	13	1.5×10^{-5}	1.83–1.85	60–64
2*e*	13	1.5×10^{-4}	1.89–1.92	78–82
2*f*	13	1.5×10^{-2}	1.95–1.97	87–91

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